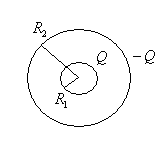
Capacitor Problems

**Problem**

What is capacitance of spherical capacitor?

**Solution**

Or consider concentric spheres with radii R1, and R2.



When we place a charge Q on the inner one, and –Q on the outer, then the **E** between the plates is:



The potential difference between the plates will then be,



and therefore the capacitance will be:

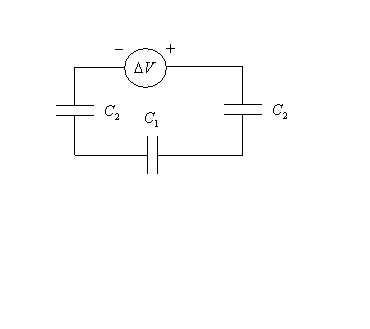


So we have,



**Problem**

What is the equivalent capacitance of the following set of capacitors? Let ΔV = 10V, C1 = 1μF, and C2 = 2μF. What is the charge on the middle capacitor? What is the potential difference across the capacitor on the right side? What is the total charge stored by the circuit?



**Solution**

All capacitors are in series. So the equivalent capacitance is given by:



Observe how the equivalent capacitance is smaller than the individual capacitances. This is always true for a series arrangement. The charge on the middle capacitor is the same as the charge on any of the three, which is the same as the charge stored on the equivalent capacitor – according to the discussion above. And this is:



The potential difference across C2 on the right is given by:

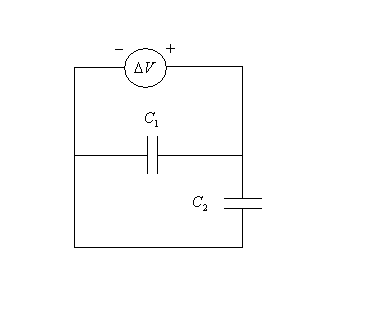


Finally, the total charge stored by the circuit is:



**Problem**

What is the equivalent capacitance of the following set of capacitors? Let ΔV = 10V, C1 = 1μF, and C2 = 2μF. Also, what is the charge stored on C1. What is the potential difference across C2? What is the total charge stored by the circuit?



The equivalent capacitance of this set of capacitors (which are in parallel) is given by:



The charge on C1 is:

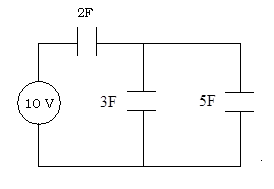


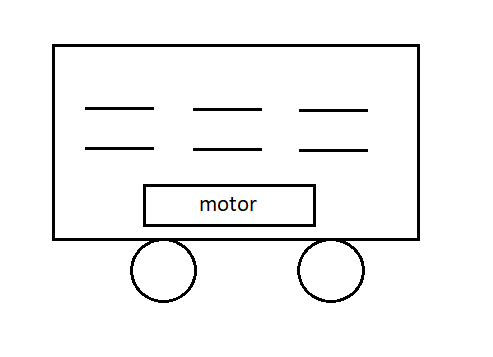
(we know that the potential difference across C1 is 10V because it is hooked up directly across the battery). The potential difference across C2 is 10V for the same reason. The total charge stored by the capacitors is:

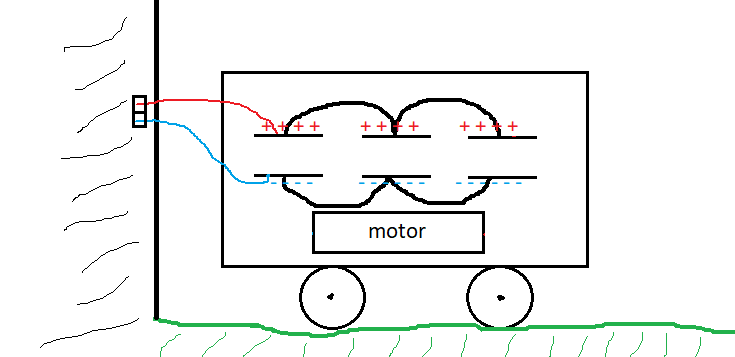


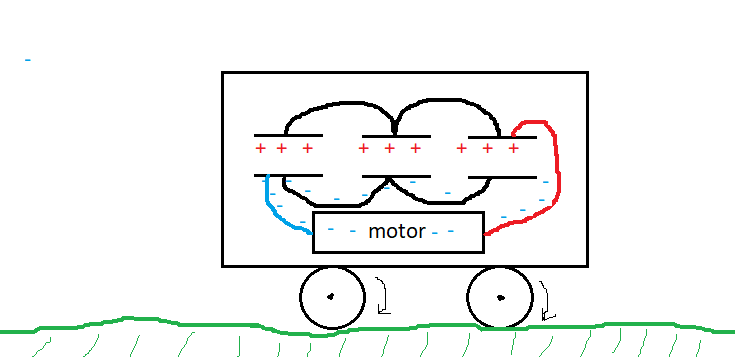
**Problem**

What is the charge across the 5F capacitor shown below?



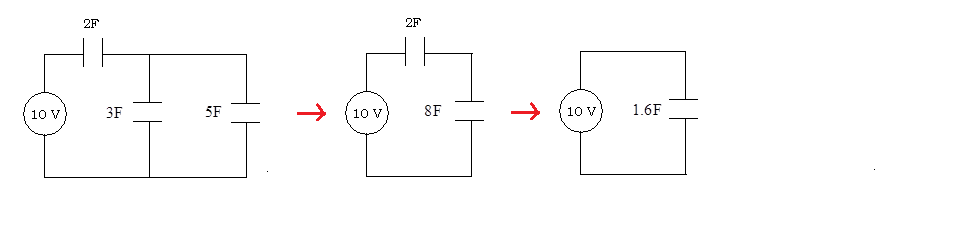






**Solution**

Let’s reduce the circuit to the equivalent capacitor step by step.

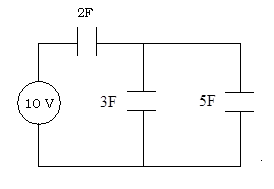


The second circuit combines 3F and 5F in parallel, and the third combines 2F and 8F in series.

Now going backwards, we expand the 1.6F capacitor in series to get the 2F and 8F. So we should calculate the charge on the equivalent capacitor. The charge is Q = CV = (1.6)(10) = 16C. And this is the charge on the 2F and 8F ones.

Next the 8F capacitor is expanded in parallel to get the 3F and 5F. So we should calculate the potential across the 8F. This is V = Q/C = 16C/8F = 2V. And this is the potential difference across the 3F and 5F ones. So then the charge on the 5F is Q = CV = (5F)(2V) = 10C.

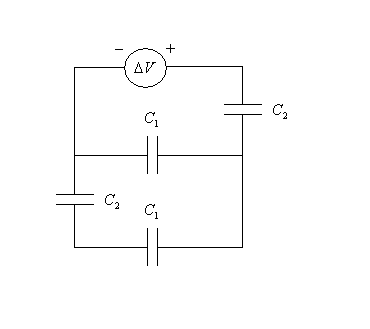
**Question 3**. What is the charge stored in the circuit below?



Q = CΔV. The equivalent capacitance is given by: C = ((3 + 5)-1 + 2-1)-1 = (1/8 + ½)-1 = (10/16)-1 = 16/10 = 1.6F. And so Q = (1.6)(10) = 16 Coulombs.

**Problem**

What is equivalent capacitance of the following circuit? What charge is stored in the circuit? What is the energy stored in the circuit? What is the charge stored on the bottom C1? Let ΔV = 10V, C1 = 1μF, and C2 = 2μF.

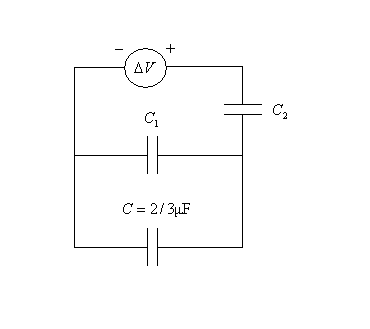


**Solution**

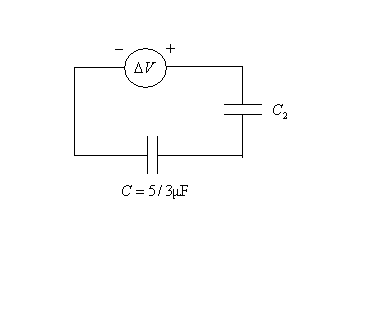
C1 and C2 are in series. Their equivalent capacitance is:



So we have the equivalent circuit,



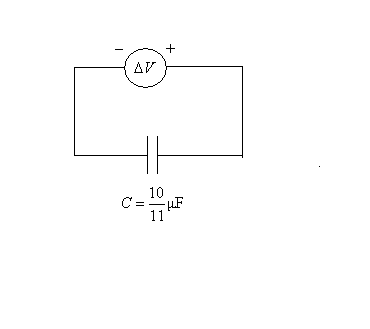
Now C and C1 are in parallel, so their equivalent capacitance is C = 1 +2/3 = 5/3μF. So now we have,



Finally, these are in series. The equivalent capacitance is:



And the equivalent circuit is:



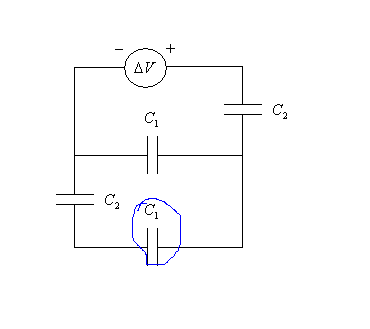
The equivalent capacitance of the circuit is therefore C = 10/11μF. The charge stored on the equivalent capacitor, and therefore the entire circuit, is:



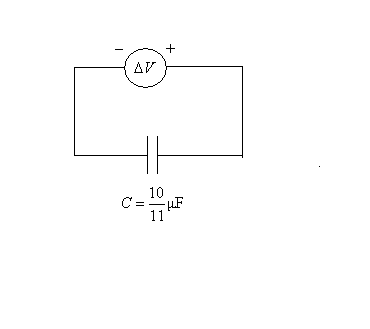
The energy stored in the circuit is:



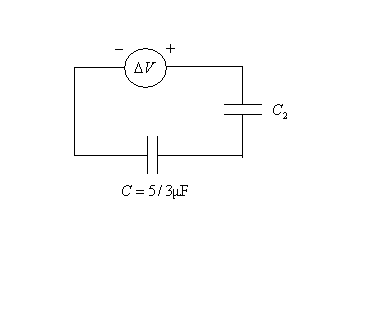
Then ask what is the charge stored in one of the particular capacitors, the bottom C1. Work backward, emphasizing that when expand capacitor in series, then charge remains the same, and when expand in parallel, the voltage remains the same. So we want the charge on that guy.



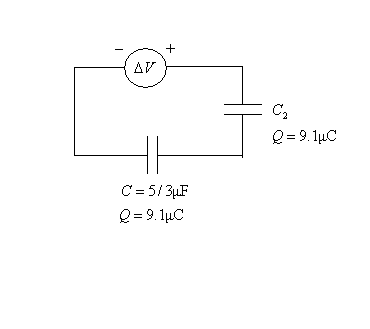
To obtain it, we start with are equivalent circuit



and work our way backwards to the capacitor in question. The circuit previous was,



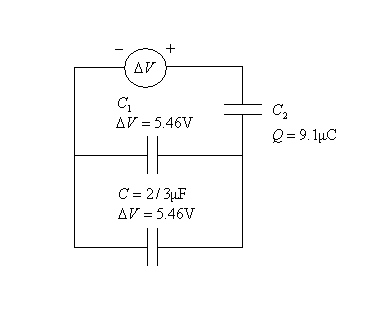
Since the 10/11 was expanded in series to get the 5/3 and C2, all three capacitors have the same charge. That charge was 9.1μC. Therefore 5/3 and C2 have 9.1μC on them.



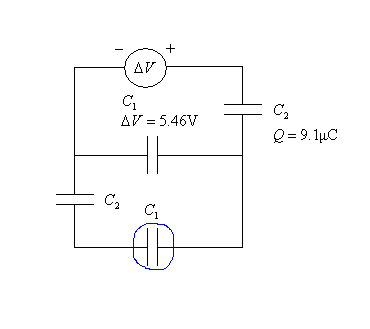
Going backwards again, C, is expanded in parallel to give us the C1 and the 2/3 below. Since C1 and 2/3 are expanded in *parallel* from C, they all share the same potential difference. That potential difference was



So we have,



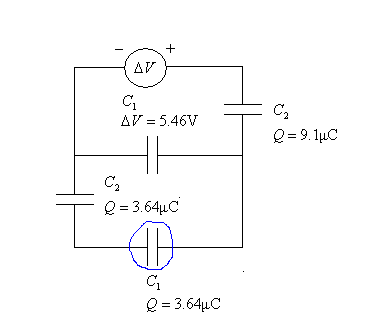
Now finally, the 2/3 is expanded in series into C2 and C1. Since it is expanded in series, the 2/3, and its series constituents C1 and C2 all have the same charge.



The charge on 2/3 is

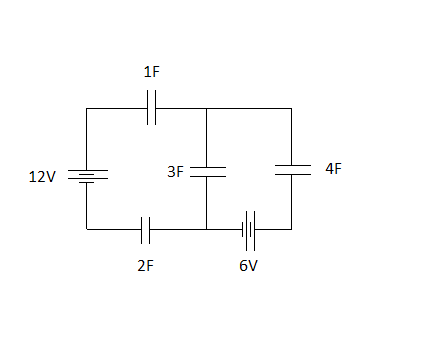


so we have,

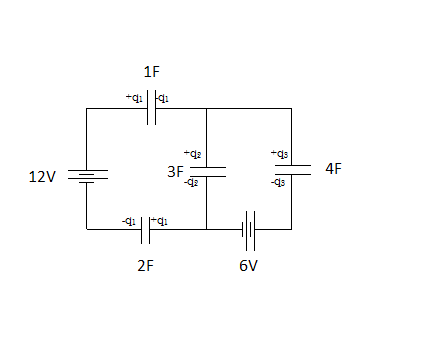


Therefore the charge on C1 = 3.64μC.

**Question 4**. What is the energy stored in the 3F capacitor?



We have to figure out the charge stored on the capacitor. So, labeling charges, and using Kirchoff’s laws:





Putting (1) into (2) and (3) we get:



Eliminating q3 from the equations,

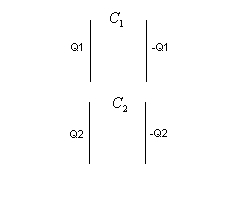


and so the energy stored on the 3F capacitor is:

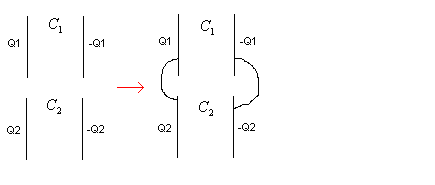


**Problem**

If you store an amount Q1 on one capacitor, C1, and Q2 on another, C2,



and connect them positive to positive in parallel,



The charges Q1 and Q2, will mix up. Some will go to the other plate, some will remain. The same will happen on the other side. Suppose Qa will reside on the first, and Qb on the second.



What will these new charges be?

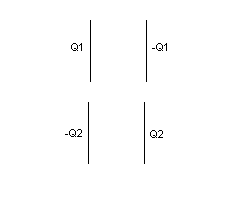
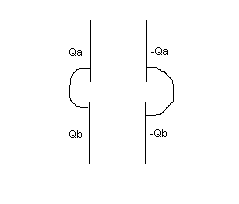
**Solution**

To determine the amounts we can assert that,

Qa + Qb = Q1 + Q2

Qa/C1 = Qb/C2

since the capacitors are in parallel they have the same V, and then we can solve these equations for the new charges. If they were connected positive to negative,

then the total charge would be |Q1 – Q2|, and we’d have,

Qa + Qb = |Q1 – Q2|

Qa/C1 = Qb/C2

Note that the plates have to end up being connected positive to positive and negative to negative. Otherwise, going around the loop you couldn’t possibly get ΔV = 0. So Qa and Qb will be positive.

**Problem**

Suppose we put 10μC on C1 = 1μF and 5μC on C2.= 2μF. If we then connect the capacitors together positive plate to positive plate, and then put a dielectric material κ = 5 in between the plates of C1, what will be the resulting charges stored on C1 and C2?

**Solution**

Charge conservation requires,



and KVL requires,



Solving…



and plugging into the first,

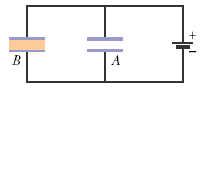


and then,



**Problem**

In the figure two parallel-plate capacitors *A* and *B* are connected in parallel across a 600V battery. Each plate has area 50cm2; the plate separations are 2mm. Capacitor *A* is filled with air; capacitor *B* is filled with a dielectric of dielectric constant κ = 3.50. Find the magnitude (in N/C) of the electric field within capacitors *A* and *B*.



**Solution**

The electric field strength is E = ΔV/d = 600V/0.002m = 300 000 N/C, for both capacitors.

**Problem 1**.

A parallel plate capacitor has dimensions A = 10cm2 and d = 2mm. It is connected to a 20V battery. And suppose we have sitting around a dielectric with dielectric constant κ = 40 and breakdown strength Emax = 150 kV/m. Assuming the capacitor is hooked up to the battery the entire time…

(a) What is *final* charge before and after the dielectric is inserted into the capacitor?

The capacitance of the capacitor is C = Aε0/d = (10)(0.01)2×8.85×10-12/0.002 = 4.42pF.

And so the charge stored at first will be Q = CΔV = (4.42)(20)pC = 88.4pC. With the dielectric inserted, the capacitance will increase by a factor of 40, and so will the charge, resulting in Q = 3.54nC.

(b) What is the *final* potential energy before and after the dielectric is inserted?

The PE = (1/2)CV2 = (1/2)

(c) What is the *final* electric field strength inside the capacitor before and after the dielectric is inserted?

The field strength in any either case is E = ΔV/d = 20/0.002 = 10kV/m.

(d) With the dielectric inserted, and the 20V battery replaced by a variable voltage battery, what is the maximum amount of charge the capacitor could store?

The maximum charge would be obtained by putting the strongest field across the capacitor, E = 150kV/m. This corresponds to a potential difference of V = Ed = 300V. And this corresponds to a charge of Q = CΔV = κC0ΔV = (40)(4.42pF)(300) = 53nC.

**Question 2.** Two 3cm-diameter electrodes with a 0.15 mm-thick sheet of Teflon between them are attached to a 12.0 V battery. (a) What is the electric field between the plates before and after the Teflon is removed.

E = ΔV/Δs = 12/0.15×10-3 = 80 000 N/C before and after.

(b) What is the charge on the plates before and after the Teflon is removed?

Q = CΔV = κ[(πr2)(8.85×10-12)/d]ΔV = (2.1)[π(1.5cm)2/0.15mm](12) = 1.05 nC (before)

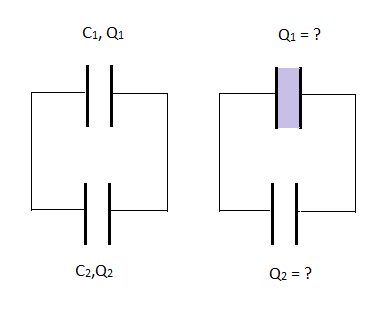
Q = CΔV = [(πr2)(8.85×10-12)/d]ΔV = (2.1)[π(1.5cm)2/0.15mm](12) = 0.502 nC (after)

(c) What is the potential energy stored before and after the Teflon is removed?

PE = (1/2)QΔV = (1/2)(1.05×10-9)(12) = 6.3 nJ (before)

PE = (1/2)QΔV = (1/2)(0.5×10-9)(12) = 3.15 nJ (after)

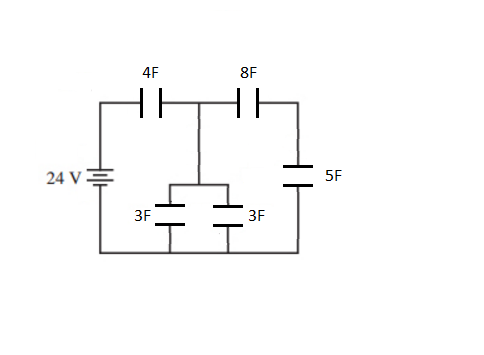
**Question 4.** Two capacitors are connected in parallel. C1 = 1F, while C2 = 2F. And Q1 = 1C, Q2 = 2C. All of a sudden (!) a dielectric material with κ = 4.6 is placed inside capacitor 1. After the charges reposition themselves, what will be the new charges on the capacitors? How much work was required to insert the dielectric?



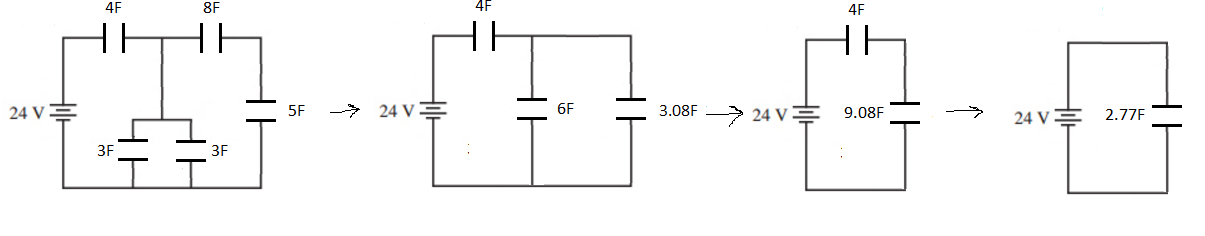
From charge conservation we have Q1 + Q2 = 1 + 2 = 3. And from (clockwise) KVL we’d have –Q1/κC1 - Q2/C2 = 0 → Q1 = Q2(κC1/C2) = (4.6∙1/2)Q2 = 2.3Q2. Substituting this back into the charge conservation equation we get: 2.3Q2 + Q2 = 3 → Q2 = 3/3.3 = 0.91 and Q1 = 3 – 0.91 = 2.09.

Work is equal to the change in energy. The energy of a capacitor is PE = (1/2)CΔV2 = Q2/2C. So PEi = (12)/(2∙1) + (22)/(2∙2) = 1.5J. And final energy is: 2.092/(2∙κ∙1) + 0.912/(2∙2) = 0.68J. So the net work was 0.68 – 1.5 = -0.72J. Work being negative indicates that the capacitor will ‘suck’ the dielectric into itself.

**Question 5.** What is the energy stored in the 8F capacitor?



We’ll reduce the circuit to an equivalent, step by step:



Then the charge on the equivalent is Q = CΔV = (2.77)(24) = 66.5C. And this is the charge on the 4 and 9.08 as well. The potential difference on the 9.08 is ΔV = Q/C = 66.5/9.08 = 7.3V, and so this is the potential difference across the 6 and 3.08 capacitors as well. The charge on the 3.08 is Q = CΔV = (3.08)(7.3) = 22.5, and so this is the charge on the 5F and 8F caps as well. So the energy stored in the 8F cap is PE = Q2/2C = (22.5)2/[2∙8] = 32J.

**Question 3.** What is the maximum energy that can be stored between two (3cm×3cm) capacitor plates seperated by 1cm, with Teflon (κ = 2.1) inserted between the plates, and a dielectric breakdown strength of 60 MV/m?



**Question 1**. A parallel plate capacitor has dimensions A = 10cm2 and d = 2mm. It is connected to a 20V battery. And suppose we have sitting around a dielectric with dielectric constant κ = 40 and breakdown strength Emax = 150 kV/m. Assuming the capacitor is hooked up to the battery the entire time…

(a) What is the *final* potential energy before and after the dielectric is inserted?

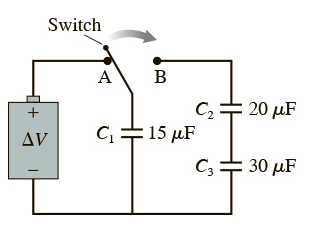
Capacitance w/o the dielectric is given by C0 = Aε0/d = (10×0.012)(8.85×10-12)/(0.002) = 4.4pF. And PE = (1/2)CV2 = (1/2)(4.4pF)(20V)2 = 0.88nJ.

PE w/ dielectric is (1/2)(κC0)V2 = 40(0.88nJ) = 35.2nJ.

(b) With the dielectric inserted, and the 20V battery replaced by a variable voltage battery, what is the maximum amount of charge the capacitor could store?

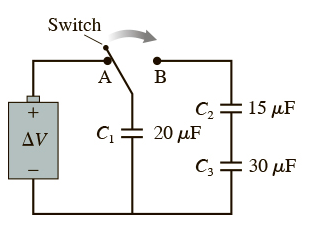
The maximum charge would be obtained by putting the strongest field across the capacitor, E = 150kV/m. This corresponds to a potential difference of V = Ed = 300V. And this corresponds to a charge of Q = CΔV = κC0ΔV = (40)(4.42pF)(300) = 53nC.

**Question 3**. Might need to check! Initially the switch is in position A and capacitors 2 and 3 are uncharged. Then the switch is flipped to position B. Afterward, what is the charge on C1 if ΔV = 75V?



With the switch in position A, the charge on C1 will be Q = CV = (15μF)(75V) = 1.13mC. When the switch is flipped to position 2, this charge will distribute itself to C2 and C3. Let’s call Q1, Q2, Q3 the charge on capacitors 1, 2, 3. Q2 = Q3 by charge conservation, and then also, by charge conservation must have Q1 + Q2 = 1.13mC. Then by Kirchoff’s voltage law we have: Q1/C1 – Q2/C2 – Q3/C3 = 0 → Q1/15μF – Q2/20μF – Q2/30μF = 0 → Q1 – 1.25Q2 = 0 → Q1 = 1.25Q2. Filling this equation into the other we get: (1.25Q2) + Q2 = 1.13mC → Q2 = 1.13mC/2.25 = 0.5mC. And so then Q3 = 0.5mC, and Q1 = 1.13mC – 0.5mC = 0.63mC.

**Question 5**. Initially the switch is in position A and capacitors 2 and 3 are uncharged. Then the switch is flipped to position B. Afterward, what is the charge on C1 if ΔV = 3kV?



With the switch in position A, the charge on C1 will be Q = CV = (20μF)(3kV) = 60mC. When the switch is flipped to position 2, this charge will distribute itself to C2 and C3. Let’s call Q1, Q2, Q3 the charge on capacitors 1, 2, 3. Q2 = Q3 by charge conservation, and then also, by charge conservation must have Q1 + Q2 = 60mC. Then by Kirchoff’s voltage law we have: Q1/C1 – Q2/C2 – Q3/C3 = 0 → Q1/20μF – Q2/15μF – Q2/30μF = 0 → 1.5Q1 – 2Q2 – Q3 = 0 → 1.5Q1 – 3Q2 = 0 → Q1 = 2Q2. Filling this equation into the other we get: 2Q2 + Q2 = 60mC → Q2 = 20mC. And so then, Q1 = 40mC.

**Question 4**. A parallel plate capacitor has dimensions A = 15cm2 and d = 4mm. It is connected to a variable voltage battery. And suppose we have sitting around a dielectric with dielectric constant κ = 8 and breakdown strength Emax =220 kV/m. With the dielectric inserted what is the maximum amount of charge the capacitor could store?

The maximum charge would be obtained by putting the strongest field across the capacitor, E = 220kV/m. This corresponds to a potential difference of V = Ed = 880 V. And this corresponds to a charge of Q = CΔV = (κAε0/d)(ΔV) = (8×15(0.01)2×8.85×10-12)/(0.004)×(880) = 24nC.